JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 30, No. 1, January–February 2007

Engineering Notes

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Global Robust Control of an Aeroelastic System Using Output Feedback

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DOI: 10.2514/1.22940

I. Introduction

AEROELASTIC systems exhibit a variety of phenomena including instability, limit cycle, and even chaotic vibration. Researchers in aerodynamics, structures, materials, and control have made interesting contributions to the analysis and control of aeroelastic systems [1]. Readers may refer to an excellent article by Mukhopadhyay [2] for the analysis and control of aeroelastic systems. A benchmark active control technology (BACT) wind-tunnel model has been designed at the NASA Langley Research Center and control algorithms for flutter suppression have been developed [3–5]. At Texas A&M a 2-degrees-of-freedom aeroelastic model has been developed and tests have been performed in a wind tunnel to examine the effect of nonlinear structural stiffness, and control systems have been designed using linear control theory, feedback linearizing technique, and adaptive control strategies [6–10]. Based on the Euler–Lagrange theory, control of an aeroelastic model has been considered [11]. The state variable and output feedback designs of [8–10] are based on adaptive control techniques. The synthesis of adaptive controllers is not simple because a large number of parameters must be updated in the dynamic feedback loop. It is also well known that unmodeled dynamics of the system can cause parameter divergence and instability in the closed-loop system. Therefore, it is interesting to design nonadaptive control systems for uncertain aeroelastic models which can be synthesized relatively easily.

The contribution of this paper lies in the design of a robust control system for the global regulation of an aeroelastic system which describes the plunge and pitch motion of a wing. The model has polynomial type structural nonlinearity and only the pitch angle is measured for feedback. It is assumed that all the system parameters are unknown to the designer, but the bounds on uncertainties are known. For the purpose of design, a first-order dynamic compensator is introduced and a "chained" structure of the aeroelastic model including the dynamic compensator is obtained by an appropriate coordinate transformation. Then using the Lyapunov stability theory, a control law for robust output regulation of the transformed system including the compensator is derived. In the closed-loop system, the controller accomplishes global robust stabilization of the aeroelastic system and system trajectories converge to the origin. Simulation results for various flow velocities and elastic axis locations are obtained which show that the control system is effective in flutter suppression in spite of the large parameter uncertainties. An attractive feature of this control system lies in its simplicity from the point of view of implementation.

II. Aeroelastic Model and Control Problem

The prototypical aeroelastic wing section is shown in Fig. 1. The governing equations of motion are given by [6–8]

$$\begin{bmatrix} m_t & m_w x_\alpha b \\ m_w x_\alpha b & I_\alpha \end{bmatrix}\begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_\alpha \end{bmatrix}\begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha(\alpha) \end{bmatrix}\begin{bmatrix} h \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ M \end{bmatrix} \tag{1}$$

where h is the plunge displacement and α is the pitch angle. In Eq. (1), m_w is the mass of the wing, m_t is the total mass, b is the semichord of the wing, I_α is the moment of inertia, x_α is the nondimensionalized distance of the center of mass from the elastic axis, k_h and $k_\alpha(\alpha)$ are the structured spring constants in plunge and pitch, c_α and c_h are the pitch and plunge damping coefficients, respectively, and L and M are the aerodynamic lift and moment. It is assumed that the quasi-steady aerodynamic force and moment are of the form

$$\begin{aligned} L &= \rho U^2 b c_{l_\alpha} s_p\{\alpha + (\dot{h}/U) + [(1/2) - a]b(\dot{\alpha}/U)\} + \rho U^2 b c_{l_\beta} s_p \beta \\ M &= \rho U^2 b c_{m_\alpha} s_p\{\alpha + (\dot{h}/U) + [(1/2) - a]b(\dot{\alpha}/U)\} \\ &\quad + \rho U^2 b c_{m_\beta} s_p \beta \end{aligned} \tag{2}$$

where U is the freestream velocity, a is the nondimensionalized distance from the midchord to the elastic axis, s_p is the span, c_{l_α} and c_{m_α} are the lift and moment coefficients per angle of attack, and c_{l_β} and c_{m_α} are lift and moment coefficients per control surface deflection β. For illustrative purposes, the function $k_\alpha(\alpha)$ is considered as given by a polynomial nonlinearity of fourth degree:

$$k_\alpha(\alpha) = k_{\alpha_0} + k_{\alpha_1}\alpha + k_{\alpha_2}\alpha^2 + k_{\alpha_3}\alpha^3 + k_{\alpha_4}\alpha^4 \tag{3}$$

Define $x = (\dot{\alpha}, \dot{h}, h)^T \in R^3$ and let the controlled output variable be $y = \alpha$. Then the aeroelatic model can be written in a state variable form as

$$\dot{x} = \bar{F}(w)x + \bar{G}(w, y)y + g(w)u \qquad \dot{y} = [1, 0, 0]x = \bar{H}x \tag{4}$$

where the control input is $u = \beta$, w denotes the vector of all the unknown parameters of the system, and $\bar{F}(w)$, $\bar{G}(w, y)$ and $g(w)$ are of the form

Received 2 February 2006; revision received 14 August 2006; accepted for publication 15 August 2006.

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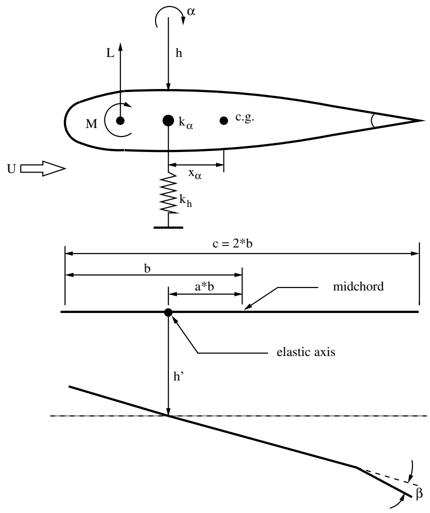


Fig. 1 Aeroelastic model.

$$\bar{F}(w) = \begin{bmatrix} \bar{f}_{11} & \bar{f}_{12} & \bar{f}_{13} \\ \bar{f}_{21} & \bar{f}_{22} & \bar{f}_{23} \\ 0 & 0 & 0 \end{bmatrix} \qquad \bar{G}(w, y) = [\bar{G}_1, \bar{G}_2, 0]^T$$

$$\bar{G}_i(w, y) = g_{i0} + g_{i1}y + g_{i2}y^2 + g_{i3}y^3 + g_{i4}y^4, \qquad i = 1, 2$$

$$g(w) = [g_1, g_2, 0]^T$$

Here T denotes the matrix transposition. It is assumed that the uncertain parameter vector $w \in W \subset R^w$, where W is a compact set. The matrices $\bar{F}(w)$, $\bar{G}(w, y)$ and $g(w)$ are easily obtained from Eqs. (1–3).

We are interested in designing a control law for global robust regulation of the pitch angle and regulation of the state vector $(x^T, \alpha)^T$ to the origin in spite of the uncertainties in the parameters. Moreover, only the pitch angle is measured for synthesis.

III. Dynamic Extension and Triangular Form

Now the design of a robust control law is considered. For global stabilization using only pitch angle feedback, it is essential to obtain a chained structure (lower triangular form) of the aeroelastic system by dynamic extension and an appropriate state transformation [12–14].

The relative degree r of the output $y = \alpha$ for the system (4) is two, because the control input u appears in the second derivative of α. As such, following [12–14], a compensator of order $r - 1$ of the form

$$\dot{\xi} = F_e \xi + G_e u \tag{5}$$

is introduced, where $\xi \in R$, $F_e = -\lambda$, λ is a positive real number, and $G_e = 1$. [The choice of $G_e = g_e(y)$, where $g_e(y)$ is a smooth function of the output is allowed.]

Consider the augmented system

$$\dot{x} = \bar{F}(w)x + \bar{G}(w, y)y + g(w)u \qquad \dot{y} = \bar{H}x$$
$$\dot{\xi} = F_e \xi + G_e u \tag{6}$$

To obtain a chained structure of Eq. (6), consider a coordinate transformation

$$z = x - D(w)\xi - h(w)y \tag{7}$$

where matrices $D(w)$ and $h(w)$ are yet to be determined. Then the system Eq. (6) has a representation of the form

$$\dot{z} = (\bar{F}(w) - h(w)\bar{H})z + \{[\bar{F} - h(w)\bar{H}]h(w) + \bar{G}(w, y)\}y$$
$$+ [\bar{F}(w)D(w) - D(w)F_e - h(w)\bar{H}D(w)]\xi + [g(w)$$
$$- D(w)G_e]u \tag{8}$$
$$\dot{y} = \bar{H}z + \bar{H}h(w)y + \bar{H}D(w)\xi \qquad \dot{\xi} = F_e\xi + G_e u$$

Now for representing system Eq. (8) in a lower triangular form, it is essential to choose the matrices D and h such that ξ and u do not appear in the derivative of z. First note that $\dot{z}$ is independent of u if one selects D such that

$$D(w) = D(w)G_e = g(w)$$

because $G_e = 1$. Defining (I denotes an identity matrix)

$$d(w) = \bar{F}(w)D(w) - D(w)F_e = [\bar{F}(w) + \lambda I]g(w)$$
$$b_0(w) = \bar{H}D(w) = \bar{H}g(w) = g_1(w)$$

one finds that $\dot{z}$ in Eq. (8) is also independent of ξ provided that h satisfies

$$h(w) = \frac{d(w)}{b_0(w)} \tag{9}$$

Substituting Eq. (9) in Eq. (8) gives the desired triangular form given by

$$\dot{z} = F(w)z + G(w, y)y \qquad \dot{y} = Hz + K(w)y + b_0(w)\xi$$
$$\dot{\xi} = -F_e\xi + G_e u \tag{10}$$

where

$$F(w) = \bar{F}(w) - \frac{d(w)}{b_0(w)}\bar{H}$$

$$G(w, y) = \left[\bar{F}(w) - \frac{d(w)}{b_0(w)}\bar{H}\right]\frac{d(w)}{b_0(w)} + \bar{G}(w, y) \qquad H = \bar{H}$$

$$K(w) = \bar{H}\frac{d(w)}{b_0(w)}$$

The transfer function of the linearized aeroelastic model obtained from Eq. (4) with its output α has stable zeros for a set of velocities U and the locations of the elastic axis. In such a case, it can be shown that the matrix $F(w)$ is Hurwitz. Setting the output of y to zero in Eq. (10) gives the zero dynamics of the form

$$\dot{z} = F(w)z \tag{11}$$

Because $F(w)$ is Hurwitz, it follows that the equilibrium state $z = 0$ of the zero dynamics is globally exponentially stable for $w \in W$. As such for stabilization of Eq. (10), it suffices to regulate the output $y(t)$ to zero.

IV. Global Robust Control Law

In this section the derivation of the control law for output regulation using the Lyapunov theory is considered [13]. The matrix $F(w)$ is Hurwitz; therefore, there exists a unique positive definite symmetric matrix $P(w)$ [denoted as $P(w) > 0$], which satisfies the Lyapunov equation

$$F^T(w)P(w) + P(w)F(w) = -I \tag{12}$$

for $w \in W$, where T denotes matrix transposition.

Step 1: Define an error variable

$$\tilde{\xi} = \xi - \alpha_1(y) \tag{13}$$

where $\alpha_1(y)$ is a stabilizing signal to be determined later. Then Eq. (10) gives

$$\dot{y} = Hz + g_1(w)[\tilde{\xi} + \alpha_1(y)] + K(w)y \tag{14}$$

For the regulation of y, consider a Lyapunov function

$$V_1 \equiv z^T P(w) z + y^2 \tag{15}$$

Then, in view of Eq. (12), its derivative along the solution of Eq. (10) is given by

$$\dot{V}_1 = -\|z\|_2^2 + 2z^T P(w) G(w, y) y + 2yHz + 2y g_1(w)(\alpha_1 + \tilde{\xi}) + 2K(w) y^2 \tag{16}$$

where $\|\cdot\|_2$ denotes the Euclidean norm. In view of Eq. (16), one chooses α_1 as

$$\alpha_1(y) = -y\gamma_1(y)[g_1(w)] \tag{17}$$

where $\gamma_1(y)$ is a smooth function.

Assumption 1: It is assumed that the sign of $g_1(w)$ is known.

We point out that the sign of $g_1(w)$ can be determined by evaluating it at some nominal values of the parameters. Then $g_1(w)$ will have same sign in a neighborhood of the nominal value of w.

Substituting Eq. (17) in (16) and rearranging gives

$$\begin{aligned}\dot{V}_1 &= -[z^T, y]\begin{bmatrix} I & -P(w)G(w,y) - H^T \\ -G^T(w,y)P(w) - H & 2[|g_1(w)|\gamma_1 - K(w)] \end{bmatrix}\begin{bmatrix} z \\ y \end{bmatrix} \\ &\quad + 2g_1 y\tilde{\xi} \\ &\equiv -[z^T, y] L_1(w, y)\begin{bmatrix} z \\ y \end{bmatrix} + 2g_1(w) y \tilde{\xi}\end{aligned} \tag{18}$$

The function $\gamma_1(y)$ is selected such that the matrix $[L_1(w, y) - \varepsilon I]$ is positive definite, where $0 < \varepsilon < 1$ is a design parameter. Following [13], it can be shown that $L_1(w, y) - \varepsilon I$ given by

$$\begin{aligned} L_1(w,y) - \varepsilon I \\ = \begin{bmatrix} (1-\varepsilon)I & -P(w)G(w,y) - H^T \\ -G^T(w,y)P(w) - H & 2[|g_1(w)|\gamma_1 - K(w)] - \varepsilon \end{bmatrix} > 0 \end{aligned}$$

is positive definite if and only if

$$2(|g_1|\gamma_1 - K) - \varepsilon - (PG + H^T)^T \frac{1}{1-\varepsilon}(PG + H^T) > 0 \tag{19}$$

It is seen that Eq. (19) is satisfied if for all $w \in W$

$$\gamma_1(y) > \frac{1}{2g_1}\left[\frac{\|P(w)G(w,y) + H^T\|_2^2}{1-\varepsilon} + 2K(w) + \varepsilon\right] \equiv \mu(y) \tag{20}$$

Where $|g_1(w)| > g_{10}$, g_{10} is a lower bound on $|g_1(w)| \in W$, and $\mu(y)$ is defined in Eq. (20). Because W is a compact set, $\mu(y)$ exists. If $\gamma_1(y)$ is chosen in this way, Eq. (19) gives

$$\dot{V}_1 \le -\varepsilon(\|z\|_2^2 + y^2) + 2g_1(w) y \tilde{\xi} \tag{21}$$

Note that if $\tilde{\xi} = 0$, then (z^T, y) converges to zero. The $\tilde{\xi}$-dependent term will be compensated in the next step.

Step 2: Now a control law is designed such that $\tilde{\xi}$ converges to zero. For this purpose, consider a modified Lyapunov function

$$V_2 = V_1 + \tilde{\xi}^2 \tag{22}$$

Taking its derivative along the solution of Eq. (10) and using Eq. (21) gives

$$\dot{V}_2 \le -\varepsilon(\|z\|_2^2 + y^2) + 2g_1(w) y\tilde{\xi} + 2\tilde{\xi}[-\lambda\tilde{\xi} + u - \dot{\alpha}_1 - \lambda\alpha_1] \tag{23}$$

The derivative of $\alpha_1(y)$ is

$$\begin{aligned}\dot{\alpha}_1 = -\frac{\mathrm{d}}{\mathrm{d}t}[y\gamma_1(y)]\mathrm{sgn}[g_1(w)] = -\mathrm{sgn}[g_1(w)]\left[\gamma_1(y) + y\frac{\partial\gamma_1}{\partial y}\right]\{Hz \\ + g_1(w)[\tilde{\xi} - \mathrm{sgn}(g_1) y\gamma_1(y)] + K(w)y\}\end{aligned} \tag{24}$$

Substituting Eq. (24) in (23) and collecting terms gives

$$\dot{V}_2 \le -\varepsilon(\|z\|_2^2 + y^2) + 2A\tilde{\xi}z + 2B\tilde{\xi}y + C\tilde{\xi}^2 + 2\tilde{\xi}u \tag{25}$$

where the row vector A and the scalar functions B and C are

$$\begin{aligned} A(w,y) &= 2\left[\gamma_1(y) + y\frac{\partial\gamma_1}{\partial y}\right] H\,\mathrm{sgn}(g_1) \\ B(w,y) &= g_1(w) - \mathrm{sgn}(g_1)\left[\gamma_1(y) + y\frac{\partial\gamma_1}{\partial y}\right][g_1\gamma_1\,\mathrm{sgn}(g_1) - K] \\ &\quad + 2\,\mathrm{sgn}(g_1)\lambda\gamma_1 \\ C(w,y) &= -2\lambda + 2\left[\gamma_1(y) + y\frac{\partial\gamma_1}{\partial y}\right] g_1\,\mathrm{sgn}(g_1) \end{aligned} \tag{26}$$

In view of Eqs. (25) and (26), one chooses a control law of the form

$$u = -\gamma_2(y, \tilde{\xi})\tilde{\xi} \tag{27}$$

where γ_2 is a smooth function. Substituting the control law Eq. (27) in (25) gives

$$\begin{aligned}\dot{V}_2 &\le -[z^T, y, \tilde{\xi}]\begin{bmatrix} \varepsilon & 0 & -A^T \\ 0 & \varepsilon & -B \\ -A & -B & 2\gamma_2 - C \end{bmatrix}\begin{bmatrix} z \\ y \\ \tilde{\xi} \end{bmatrix} \\ &\equiv -x_a^T L_2(w, y, \tilde{\xi}) x_a \end{aligned} \tag{28}$$

where $x_a = (z^T, y, \tilde{\xi})^T \in R^5$ and the matrix L_2 is defined in Eq. (28).

Let us choose γ_2 such that

$$L_2(w, y, \tilde{\xi}) - \frac{\varepsilon}{2} I > 0 \tag{29}$$

In view of (28), $L_2(w, y, \tilde{\xi}) - \frac{\varepsilon}{2} I$ is positive definite if and only if

$$2\gamma_2 - C - \frac{\varepsilon}{2} > \frac{2}{\varepsilon}\|[A(w,y), B(w,y)]^T\|_2^2 \tag{30}$$

We point out that, in general, Eq. (30) can be satisfied by a choice of γ_2 which is a function of $\tilde{\xi}$ and y. However, for simplicity, we shall choose only y-dependent function γ_2. Then Eq. (30) is satisfied if

$$\gamma_2(y) > \frac{1}{2}\max\left\{C + \frac{\varepsilon}{2}\|[A(w,y), B(w,y)]^T\|_2^2\right\} + \frac{\varepsilon}{4} \equiv \mu_2(y) \tag{31}$$

For a choice of $\gamma_2(y)$, according to (31), Eq. (28) gives

$$\dot{V}_2 < -\frac{\varepsilon}{2}(\|z\|_2^2 + y^2 + \tilde{\xi}^2) \tag{32}$$

Because V_2 is a positive definite function of x_a and $\dot{V}_2$ is negative definite quadratic function of x_a, it follows from the Lyapunov stability theory (see [15], p. 173) that the origin $x_a = 0$ is globally asymptotically stable.

In view of the derivation of this section, we state the following theorem.

Theorem 1: Consider the aeroelastic system Eq. (4) and the compensator Eq. (5) and suppose that the system (4) with output $y = \alpha$ is minimum phase. Then the control law Eq. (27) accomplishes global robust regulation of the pitch angle to zero and the equilibrium state ($x^T = 0$, $y = 0$, $\xi = 0$) of the augmented system Eq. (6) is globally asymptotically stable.

Proof: It has already been shown that $(z^T, y, \tilde{\xi})^T$ is globally asymptotically stable. Thus as y converges to zero, $\alpha_1(y)$ tends to zero. Then in view of Eq. (13), convergence of $\tilde{\xi}$ and $\alpha_1(y)$ to zero implies that ξ tends to zero as well. Now in view of the coordinate transformation $z = x - D(w)\xi - h(w)y$, it follows that $x(t)$ asymptotically converges to zero. As such the equilibrium point $(x^T, y, \xi) = 0$ of the system Eq. (6) is globally asymptotically stable.

To this end, it is appropriate to discuss the implementation issue of this control system. Theorem 1 provides a sufficient condition for stability in the closed-loop system. For the synthesis of the control law Eq. (27), the functions $\gamma_i(y)$ satisfying inequalities Eqs. (20) and (31) are computed using the bounds on the uncertain parameter vector w. Substituting for $\tilde{\xi}$ in Eq. (27), the nonlinear control law takes the form

$$u = -\gamma_2(y)\{\xi + y\gamma_1(y)\,\mathrm{sgn}[g_1(w)]\} \tag{33}$$

Of course, the control law Eq. (27) has a simple structure. However, still a further simplification in the control law is possible if one sets these functions $\gamma_i(y)$ to certain overestimated constants values. However, for the choice of some constant values of $\gamma_i(y)$, one can expect only semiglobal stability and only a finite region of stability exists. With such a simpler control law, the trajectories converge to the origin only if the initial conditions lie in the region of stability. But the region of stability can be enlarged by taking larger values of $\gamma_i(y)$. Indeed in the next section, it will be seen that it is possible to accomplish flutter control using this simplified control law with constant gains $\gamma_i(y)$.

V. Simulation Results

This section presents the results of digital simulation. The initial conditions chosen are $x(0) = [0, 0, h(0) = 0.01(m)]^T$, $\alpha(0) = 10$ deg, and $\xi(0) = 0$. The parameters of the aeroelastic model are taken from [6–8] and are collected in the Appendix. The dynamic compensator's parameter is set to $F_e = -\lambda = -5$. In view of Theorem 1, we note that the inequalities Eqs. (20) and (31) yield feedback gains γ_i only to satisfy the sufficient (not necessary) conditions for stability in the closed-loop system. Therefore, for simplicity, γ_i are selected as $\gamma_1 = 4$ and $\gamma_2 = 5$. These values of γ_i have been obtained by observing the simulated responses. To save space, results are presented for values of $U = 25$ m/s and $a = -0.6847$. Computing the zeros of the linearized model 1, one finds that the transfer function has stable zeros, and the parameter $b_0(w) = g_1(w)$ is negative. Thus the aeroelastic system is minimum phase.

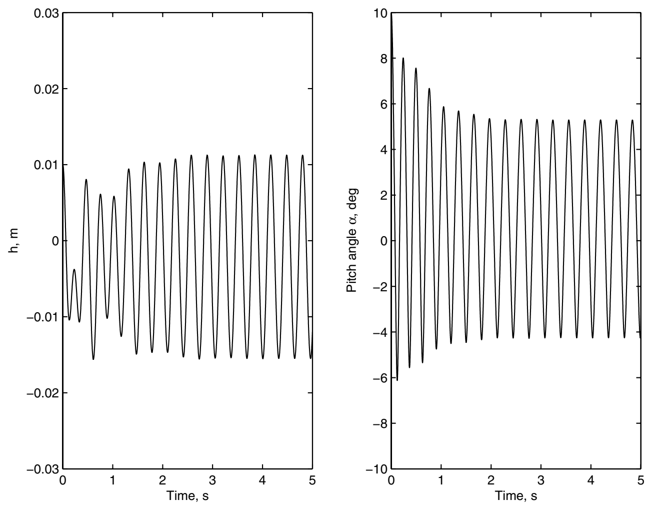
The open-loop responses are shown in Fig. 2 for $U = 25$ m/s and $a = -0.6847$. We observe limit cycle oscillations of the pitch angle and the plunge displacement.

Case A1: Closed-loop control: $U = 25$, $a = -0.6847$

The closed-loop system [Eqs. (4), (5), and (27)] is simulated for the freestream velocity $U = 25$ m/s and $a = -0.6847$. For simulation, control surface deflection has been limited to 30 deg. The responses are shown in Fig. 3. We observe that after initial oscillatory transients, the state vector converges to the origin. Control input saturates in the initial transient phase. The response time is of the order of less than 2.5 s. Interestingly, a simple control law using only the pitch angle feedback accomplishes flutter suppression.

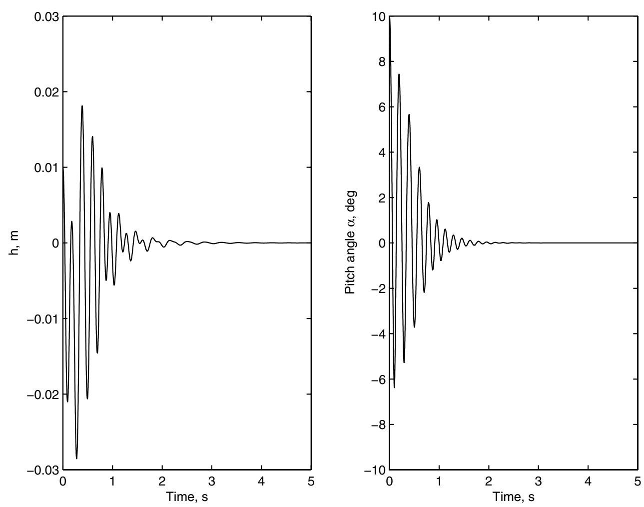
Case A2: Closed-loop control with observation noise: $U = 25$, $a = -0.6847$

To examine the sensitivity of the controller, simulation is done using the sensor noise. A white Gaussian noise of mean value zero and unit variance is applied to a first-order filter with its pole at -9 to generate a sensor noise, θ_n. In the control law, Eq. (33), instead of θ, the signal $(\theta + \theta_n)$ is used for feedback. The plot of the sensor noise is shown in Fig. 4. We observe that in the presence of measurement noise (Fig. 4), the pitch and plunge displacement do not converge to zero as expected. However, the perturbations in the state variables are not significant. Of course, these perturbations will increase with the magnitude of the sensor noise. Simulation has been done for other values of U and a. It has been found that the controller accomplishes flutter control.



a) b)

Fig. 2 Open-loop responses showing limit cycle oscillations: a) plunge displacement, b) pitch angle.



a) b)

Fig. 3 Closed-loop responses: a) plunge displacement, b) pitch angle.

To this end, a comparison of this control system with adaptive and nonadaptive feedback linearizing control systems of [6–10] is made. We may point out that compared to the indicated controllers of [6–10], the control law of this paper is synthesized easily. The simulated responses of Ko et al. [6] are smoother, but for the design it is assumed that the parameters are known. The feedback adaptive controllers of [7,8] accomplish flutter control but require full state feedback. Adaptive output feedback designs have been used in [9,10], but the oscillatory responses exit for longer period. Of course, sensor noise has not been considered in these papers.

VI. Conclusions

In this paper, global robust control law for a prototypical aeroelastic wing section with structural pitch nonlinearity using a single control surface and pitch angle measurement was derived. A simplification in this control law was obtained by replacing the pitch angle dependent gains by constant gains. However, the simplified control law can yield only semiglobal stability. Simulation results using the simplified controller showed that flutter suppression can be accomplished in spite of the uncertainties in the system and the effect of sensor noise is minor.

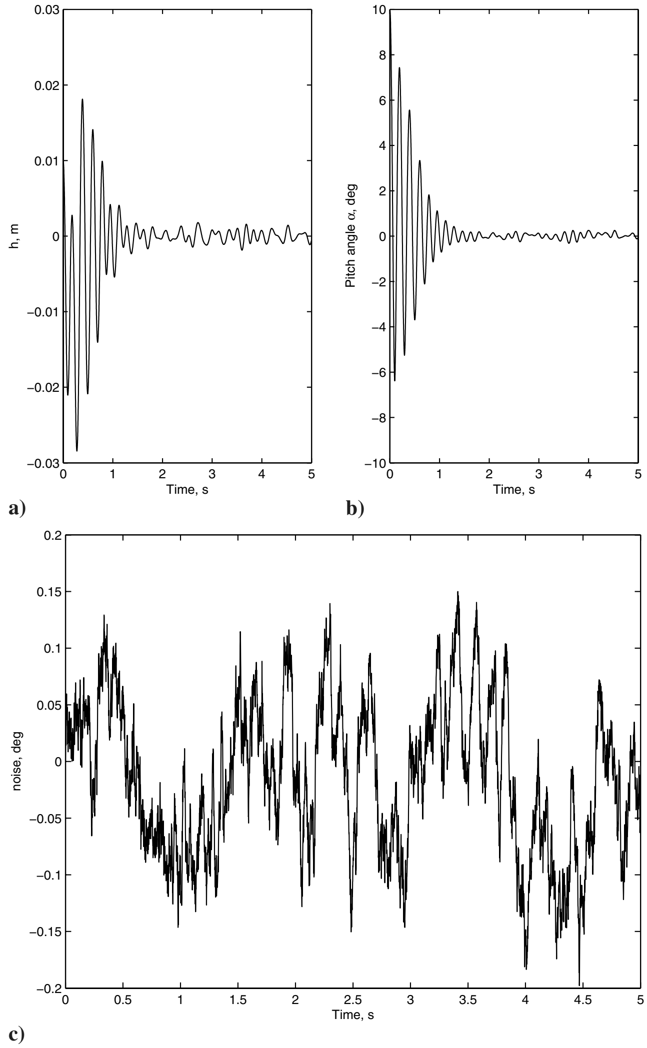


Fig. 4 Closed-loop responses: a) plunge displacement, b) pitch angle, c) noise.

Appendix: System Parameters

The system parameters are

$$b = 0.135\ \mathrm{m} \qquad m_w = 2.049\ \mathrm{kg} \qquad c_h = 27.43\ \mathrm{N\cdot s/m}$$

$$c_\alpha = 0.036\ \mathrm{N\cdot s} \qquad \rho = 1.225\ \mathrm{kg/m^3} \qquad c_{l_\alpha} = 6.28$$

$$c_{l_\beta} = 3.358 \qquad c_{m_\alpha} = (0.5 + a)c_{l_\alpha} \qquad c_{m_\beta} = -0.635$$

$$m_t = 12.387\ \mathrm{kg} \qquad I_\alpha = 0.0517 + m_w x_\alpha^2 b^2\ \mathrm{kg\cdot m^2}$$

$$x_\alpha = [0.0873 - (b + ab)]/b \qquad k_h = 2844.4\ \mathrm{N/m}$$

$$k_\alpha = 2.82[1 - 22.1\alpha + 1315.5\alpha^2 - 8580\alpha^3 + 17289.7\alpha^4]\ \mathrm{N\cdot m/rad}$$

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